Matrices de Fourier aléatoires et asymptotique de spectres d'opérateurs.

Abstract. Given two probability laws on $\mathbb{R}^d$, called P and Q, we are interested in the asymptotic behavior of the spectrum of the operator T_m on $L^2(P)$, defined by

$$T_m f(x) = \int_{\mathbb{R}^d} \widehat{Q}(m(x-y)) f(y) dP(y)$$

for m tending to ∞. The operator T_m is positive definite with trace 1, and its spectrum is a sequence of non negative numbers of sum 1.

This spectrum is explicitly known in the Gaussian case. It has been very much studied when P and Q are uniform distributions on two symmetric intervals, in relation with the study of band limited functions. In particular, one knows that, for two uniform laws on $(-1/2, +(1/2)$, the number of degrees of freedom of T_m, which is defined as

$$\deg_\infty(T_m, \varepsilon) = \max\{s; \lambda_s(T_m) \geq \varepsilon\}$$

is approximately m with a very small error for m very large, whatever the value taken by $\varepsilon \in (0,1)$.

We consider generalizations of these properties for general probability laws, and in particular for uniformly distributed on two measurable sets of finite measure in $\mathbb{R}^d$.

This may be used to prove analogous properties for random Fourier matrices, that is, $n' \times n$ matrices A whose (j,k) entries are $\exp(2i\pi m\langle X_j, Y_k\rangle)$, with two independent sequences, X_j and Y_k, of random vectors, and where m is a real number that measures the sizes of oscillations.