Some Combinatorial Geometry of Lattice Points of the Standard 3-Simplex Δ_3.

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Abstract: By associating certain discrete objects $\mathcal{C}^3_{(3,r)_{r\geq 1}}$ wirh Grassmannians $\mathrm{Gr}(3,3+r)_{r\geq 1}$, we study the geometry of the lattice points of dilations $r\Delta_3$ of the standard 3-simplex. The elements of these discrete objects parametrized by the positive integers $r\in\mathbb{Z}_{\geq 1}$ are counted by tetrahedral polynomials $\mathrm{T}(r)\in\mathbb{Z}[t]$. We give a generating function of these polynomials as r grows. We show that the weights of these elements are precisely the lattice points of $r\Delta_3$ and the β- partition λ of each of these weights is an indexing partition $\lambda\subseteq 3\times r$ of the Schubert variety X_λ in $\mathrm{Gr}(3,3+r)$. Lastly, we show that the Hilbert series $\mathrm{H}(R_{\mathcal{C}^3_{(3,r)}},t)$ of quotient ring $R_{\mathcal{C}^3_{(3,r)}}$ identified with $\mathcal{C}^3_{(3,r)}$ coincides with the Poincaré polynomial $\mathrm{P}(\mathrm{Gr}(3,3+r),t)$ of the Grassmannian $\mathrm{Gr}(3,3+r)$.