Reliable Computation of the Singularities of the Projection in $\mathbb{R}^3$ of a Generic Surface of $\mathbb{R}^4$

Sény Diatta[1,2], Guillaume Moroz[2], and Marc Pouget[2]

[1] University Assane Seck of Ziguinchor, Sénégal, senydiatta@gmail.com
[2] Université de Lorraine, CNRS, Inria, LORIA, F-54000 Nancy, France, Firstname.Name@inria.fr.

Abstract

Computing efficiently the singularities of surfaces embedded in $\mathbb{R}^3$ is a difficult problem, and most state-of-the-art approaches only handle the case of surfaces defined by polynomial equations. Let F and G be C^∞ functions from $\mathbb{R}^4$ to $\mathbb{R}$ and $\mathcal{M} = \{(x, y, z, t) \in \mathbb{R}^4 \,|\, F(x, y, z, t) = G(x, y, z, t) = 0\}$ be the surface they define. Generically, the surface $\mathcal{M}$ is smooth and its projection Ω in $\mathbb{R}^3$ is singular. After describing the types of singularities that appear generically in Ω, we design a numerically well-posed system that encodes them. This can be used to return a set of boxes that enclose the singularities of Ω as tightly as required. As opposed to state-of-the art approaches, our approach is not restricted to polynomial mapping, and can handle trigonometric or exponential functions for example.